JOURNAL OF THERMOPHYSICS AND HEAT TRANSFER
Vol. 18, No. 2, April–June 2004

Analysis of Thermal Resistance of Orthotropic Materials Used for Heat Spreaders

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The general solution to the thermal resistance through a rectangular orthotropic thermal spreader depends on several dimensionless geometric and thermal parameters. The solution is presented graphically for several values of the length-to-thickness thermal conductivity ratio, the Biot number, and the nondimensional width of the applied heat flux, which can be readily applied to engineering applications for electronic component design. The graphs are applicable to a wide range of spreader materials that include aluminum, K1100/epoxy, and K1100/C–C graphite composites. A methodology is presented to assist thermal engineers in designing thermal management systems for electronic components. The advantages of the use of high-conductivity composite materials over ordinary engineering materials in terms of weight savings are illustrated by three examples, in which a comparison is made between aluminum and composites for use as heat spreader materials in typical electronic packaging heat transfer applications.

Nomenclature

a	=	width of applied heat region
Bi	=	Biot number, hc/k_y
b	=	width of plate
c	=	plate thickness
e	=	relative error
H	=	c/b per unit depth
h	=	convective heat transfer coefficient
k_x	=	thermal conductivity in the x direction
k_y	=	thermal conductivity in the y direction
k^2	=	ratio of plate conductivities, k_y/k_x
L	=	plate length normal to cross section
q_t	=	total heat transfer
q_0	=	one-dimensional heat flux distribution
R	=	general dimensionless thermal resistance
R_r	=	reference thermal resistance per unit depth
$\bar{R}$	=	thermal resistance of plate per unit depth
T	=	temperature field
T_f	=	sink temperature of convective boundary case
T_r	=	reference temperature, q_0c/k_y
T_0	=	sink temperature of isothermal boundary case
W	=	a/b
x, y	=	Cartesian coordinates
α	=	nondimensional plate thickness, a/c
β	=	H^2/k^2
ε	=	nondimensional width, b/c
ζ	=	nondimensional coordinate, x/c
η	=	nondimensional coordinate, y/c
θ	=	nondimensional temperature, $(T - T_0)/T_r$ for isothermal bottom boundary case, $(T - T_f)/T_r$ for convective bottom boundary case
λ_n	=	eigenvalue

Subscripts

cs	=	convective sink
is	=	isothermal sink
n	=	a function of the integer n
r	=	reference

Received 19 May 2003; revision received 16 October 2003; accepted for publication 17 October 2003.

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Introduction

CONSIDERABLE attention has been given in recent years to the development of low-weight and high-conductivity orthotropic materials in the power semiconductor industry to accommodate high thermal density electronics.[1,2] Advanced graphite materials such as K1100, P120, and carbon–carbon composites are well-known examples of such materials. The thermal response and thermal resistance of composites are important factors in electronic cooling applications.[3] The mathematical complexity in the calculation of the thermal problem for orthotropic media provides only a very limited number of available solutions. The purpose of this paper is to examine the thermal response and determine the thermal resistance of an orthotropic plate subjected to a uniform heat flux. The investigation applies to typical conduction plates (also called thermal doublers) that are used to spread the heat of high-powered electronics in space, as well as to terrestrial electronic components cooled by convection.

The configuration under consideration (Fig. 1) is a two-dimensional rectangular spreader with thermal orthotropy. One side of the spreader is subjected to a uniform heat flux distributed over a portion of its boundary. The opposite side of the plate dissipates heat to an environment/sink at constant temperature through convection or is maintained as an isothermal boundary. In the application of spacecraft thermal doublers, the convective boundary condition simulates the contact resistance between the plate and the rest of the spacecraft. In the case of a convectively cooled electronic device, this boundary condition simulates the thermal sink provided by the environmental cooling fluid. The remaining two sides of the plate are either insulated from its surrounding or contacted with an isothermal or convective sink. One of these adiabatic sides might also simulate a symmetry condition, whereas the other side is exposed to the outside environment.

The effective thermal resistance is presented graphically for several values of the length-to-thickness conductivity ratio, the Biot number, and the nondimensional width of the applied heat flux region. The study applies to a wide range of spreader materials that include aluminum, K1100/epoxy, and K1100/C–C graphite composites. The solution from this study can be readily applied to actual engineering designs.

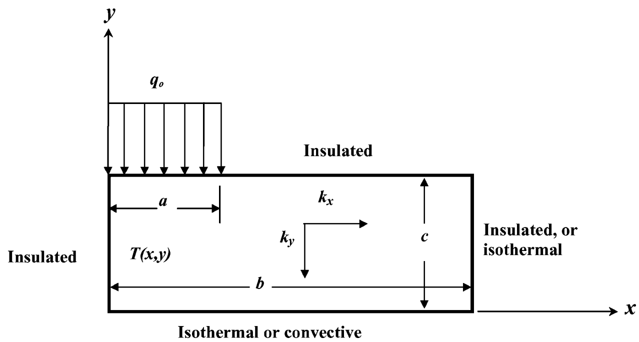

Fig. 1 Schematic of rectangular, orthotropic plate.

As mentioned, the configuration investigated simulates the cooling of an integrated circuit device.[4] The durability and long-term performance of semiconductor devices are strongly dependent on their operating junction temperatures.[5,6] As a result, knowledge of the thermal responses and thermal resistances of mounting plate materials[7,8] are of great importance to electronic designers, and applications of the present solution are discussed hereafter. In addition, the mathematical and physical limits of the solution are discussed.

Analysis

Governing Equations

The medium under consideration (Fig. 1) is a rectangular plate with thermal orthotropy. The plate conductivity is specified completely by conductivity values for the x and y directions. One side of the plate is subjected to a heat flux distribution over a portion of the boundary, whereas the opposite side dissipates heat to an environment at uniform temperature T_0 or T_f. The remaining two sides are insulated from the surroundings. Figure 1 shows the geometry and the coordinates for the two-dimensional problem.

Physical properties of the convective heat sink and the heat source, which include thermal conductivity and convection coefficient, are assumed constant. Consequently, the governing equation for the dimensionless two-dimensional steady-state heat conduction problem can be stated as

$$\frac{\partial^2\theta}{\partial\zeta^2} + k^2\frac{\partial^2\theta}{\partial\eta^2} = 0, \qquad 0 < \zeta < \varepsilon, \qquad 0 < \eta < 1 \tag{1}$$

$$\frac{\partial\theta}{\partial\eta} = \begin{cases} 1, & 0 \le \zeta \le \alpha, \quad \eta = 1 \\ 0, & \alpha < \zeta \le \varepsilon, \quad \eta = 1 \end{cases} \tag{1a}$$

$$\frac{\partial\theta}{\partial\zeta} = 0, \qquad \zeta = 0, \qquad 0 < \eta < 1 \tag{1b}$$

The remaining surfaces are subjected to various boundary conditions that include an insulated surface, isothermal sink, or convective sink. These boundary conditions are listed here for reference: 1) case 1, insulated right and isothermal bottom surfaces

$$\theta = 0, \qquad 0 \le \zeta \le \varepsilon, \qquad \eta = 0 \tag{2a}$$

$$\frac{\partial\theta}{\partial\zeta} = 0, \qquad \zeta = \varepsilon, \qquad 0 \le \eta \le 1 \tag{2b}$$

2) case 2, insulated right and convective bottom surfaces

$$\frac{\partial\theta}{\partial\eta} = Bi\theta, \qquad 0 \le \zeta \le \varepsilon, \qquad \eta = 0 \tag{3a}$$

$$\frac{\partial\theta}{\partial\zeta} = 0, \qquad \zeta = \varepsilon, \qquad 0 \le \eta \le 1 \tag{3b}$$

and 3) case 3, isothermal right and isothermal bottom surfaces

$$\theta = 0, \qquad 0 \le \zeta \le \varepsilon, \qquad \eta = 0 \tag{4a}$$

$$\theta = 0, \qquad \zeta = \varepsilon, \qquad 0 \le \eta \le 1 \tag{4b}$$

Table 1 Eigenvalues for Eqs. (5) and (6)

Case	Eigenvalue, λ_n
1	$n\pi/\varepsilon$
2	$n\pi/\varepsilon$
3	$[(1+2n)\pi]/2\varepsilon$

with the Biot number Bi defined in terms of the plate conductivity k_y.

Temperature Field

The governing equation is a nonhomogeneous, second-order differential equation. The conventional method of separation of variables can be applied directly in such problems. A well-established solution procedure for this type of problem is available in the literature.[9,10] After the procedure, the general solution of Eq. (1) in terms of an infinite series is as follows: isothermal bottom sink,

$$\theta(\zeta,\eta) = \frac{\alpha}{\varepsilon}\cdot\eta + 2\sum_{n=1}^{\infty}\frac{k\cdot\sin(\lambda_n\alpha)\cdot\cos(\lambda_n\zeta)\cdot\sinh[(\lambda_n/k)\eta]}{\varepsilon\cdot\lambda_n^2\cdot\cosh(\lambda_n/k)} \tag{5}$$

and convective bottom sink,

$$\theta(\zeta,\eta) = \frac{\alpha}{\varepsilon}\cdot\left(\frac{1}{Bi}+\eta\right) + 2\sum_{n=1}^{\infty}\frac{k\cdot\sin(\lambda_n\alpha)\cdot\cos(\lambda_n\zeta)\cdot\Phi}{\varepsilon\cdot\lambda_n^2} \tag{6}$$

where

$$\Phi = \frac{\lambda_n\cdot\cosh[(\lambda_n/k)\eta] + k\cdot Bi\cdot\sinh[(\lambda_n/k)\eta]}{\lambda_n\cdot\sinh(\lambda_n/k) + k\cdot Bi\cdot\cosh(\lambda_n/k)} \tag{6a}$$

The solution given by Eq. (5) is applicable for cases 1 and 3, whereas Eq. (6) is applicable for case 2. However, the eigenvalues in these equations vary with the right-hand side boundary conditions as provided in Eqs. (2–4). The respective eigenvalues λ_n for various cases are listed in Table 1.

Thermal Resistance

Definition

Based on general engineering practice,[11] the thermal resistance is defined as

$$\bar{R} = \Delta T_{\max}/q_t \tag{7}$$

where $\Delta T_{\max}$ is the maximum temperature difference across the spreader thickness and q_t is the total energy input. For the isothermal bottom boundary case, Eq. (7) becomes

$$\bar{R}_{\text{is}} = (T - T_0)/q_0 a = (\theta_{\eta=1})(T_r)/q_0 a \tag{8}$$

where

$$T_r = q_0 c/k_y$$

If we now define R_r as the reference resistance of the spreader in which the same energy input q_0 is applied across the entire top surface ($y = c$), Fourier law gives

$$q_t = k_y A_{y=c}(\Delta T/\Delta y) \tag{9}$$

By the application of knowledge that $A_{y=c} = b$ (unit depth), $\Delta T = T - T_0$, and $\Delta y = c$,

$$R_r = \Delta T/q_t = H/k_y \tag{10}$$

Hence, the generalized resistance for the isothermal sink case can now be written as

$$R_{\text{is}} = \bar{R}_{\text{is}}/R_r = (1/W)\theta_{\text{is}}(0,1) \tag{11}$$

A similar expression for the resistance can be developed for the case with a convective bottom boundary condition. Given that

$$\bar{R}_{cs} = \{[\theta_{cs}(0,1) - \theta_{cs}(0,0)] \cdot T_r\}/q_0 a \tag{12}$$

the resistance equation becomes

$$R_{cs} = \bar{R}_{cs}/R_r = [\theta_{cs}(0,1) - \theta_{cs}(0,0)]/W \tag{13}$$

Solutions

Based on the definition just discussed, the thermal resistance for the three configurations is given here for the isothermal bottom sink,

$$R_{is} = 1 + \frac{2H}{W}\sum_{n=1}^{\infty}\frac{\sin(\lambda_n\alpha)\cdot\sinh(\lambda_n/k)}{\lambda_n^2\cdot\cosh(\lambda_n/k)} \tag{14}$$

and convective bottom sink,

$$R_{cs} = 1 + \frac{2H}{W}\sum_{n=1}^{\infty}\frac{k\cdot\sin(\lambda_n\alpha)\cdot\Theta}{\lambda_n^2} \tag{15}$$

where

$$\Theta = \frac{\lambda_n\cdot\cosh(\lambda_n/k) + k\cdot Bi\cdot\sinh(\lambda_n/k) - \lambda_n}{\lambda_n\cdot\sinh(\lambda_n/k) + k\cdot Bi\cdot\cosh(\lambda_n/k)} \tag{15a}$$

Through substitution of the eigenvalue solutions and definition of a new variable $\beta = H^2/k^2$, the thermal resistance reduces to the following equations: case 1,

$$R_{is} = 1 + 2\sum_{n=1}^{\infty}\frac{\sin(n\pi W)\cdot\tanh\left(n\pi\sqrt{\beta}\right)}{\left(W\sqrt{\beta}\right)(n\pi)^2} \tag{16}$$

case 2,

$$R_{cs} = 1 + 2\sum_{n=1}^{\infty}\frac{\sin(n\pi W)\cdot\Phi}{\left(W\sqrt{\beta}\right)(n\pi)^2} \tag{17}$$

where

$$\Phi = \frac{\cosh\left(n\pi\sqrt{\beta}\right) + \left(Bi\big/n\pi\sqrt{\beta}\right)\sinh\left(n\pi\sqrt{\beta}\right) - 1}{\sinh\left(n\pi\sqrt{\beta}\right) + \left(Bi\big/n\pi\sqrt{\beta}\right)\cosh\left(n\pi\sqrt{\beta}\right)} \tag{17a}$$

and case 3,

$$R_{is} = 1 + 2\sum_{n=1}^{\infty}\frac{\sin\{[(1+2n)/2]\pi W\}\cdot\tanh\left\{[(1+2n)/2]\pi\sqrt{\beta}\right\}}{\left(W\sqrt{\beta}\right)\{[(1+2n)/2]/\pi\}^2} \tag{18}$$

Reduction of these equations defines the thermal resistance as a function of only W and β.

Solution Method

Solutions of the nondimensionl thermal resistance represented by Eqs. (16–18) are in the form of an infinite series. The iteration procedure used in the calculation process terminates based on preselected criteria with accuracy e. The stopping criterion for this series is based on a relative error test commonly used in engineering and scientific computations. If $R_{n=1}$ and R_n are two successive partial sums for the thermal resistance, then the relative error test can be stated as

$$\text{relative error} = \frac{|R_{n+1} - R_n|}{|R_{n+1}|} < e \tag{19}$$

The iteration process for the thermal resistance reaches closure when the error criterion is $e \leq 10^{-6}$.

Results

As mentioned earlier, the purpose of this study is to find the solution to the thermal resistance of a rectangular orthotropic thermal spreader for various boundary conditions. Results are presented graphically to show dependence on dimensionless variables and to provide a database for electronic component design. The temperature response and thermal resistances for various heat sink materials are important factors in electronic cooling applications. This paper determines the thermal resistance of an orthotropic plate subjected to the heat dissipation of an electronic box or a semiconductor chip. It is impossible to analyze each material available in the commercial market. Results for two composite materials and one isotropic material are selected to show the design benefit of modern composite materials. The thermal resistance for other materials can be easily generated from the results that are provided in this paper. Orthotropic materials such as K1100 graphite/epoxy and K1100 graphite/C–C are selected because they are widely used in the space industry for electronic box mounting applications. Their physical properties are listed in Table 2.

Figure 2 presents the nondimensional thermal resistance R_{is} as a function of the heated footprint W and β, which depends on the aspect ratio H and the ratio of conductivities k^2 for case 1 (insulated right and isothermal bottom surfaces).

Based on the material physical properties given in Table 2, the square of the length-to-thickness conductivity ratios for K1100 graphite/epoxy and K1100 graphite/C–C are 0.0074 and 0.1143, respectively. These values are represented in Fig. 2 as the denominator of the calculated variable β.

The general dimensionless thermal resistance is defined as the ratio of the thermal resistance of plate (based on the width of the imposed heat flux) to the reference thermal resistance (full width of the spreader). The plate thermal resistance equals the reference thermal resistance when the width a of the input heat flux has the same width as the heat spreader b. Therefore, the general dimensionless thermal resistance converges to unity under this special configuration as $W(=a/b) = 1$. The solution of the dimensionless thermal resistance at $W = 1$ for all values of β converges to unity,

Table 2 Material physical properties

Material	In-plane conductivity k_x, W/m · K	Out-of-plane conductivity k_y, W/m · K	Density, kg/m^3
Pure Al	221.5	221.5	2770
K1100 graphite/epoxy	270	2	1800
K1100 graphite/C–C	350	40	1800

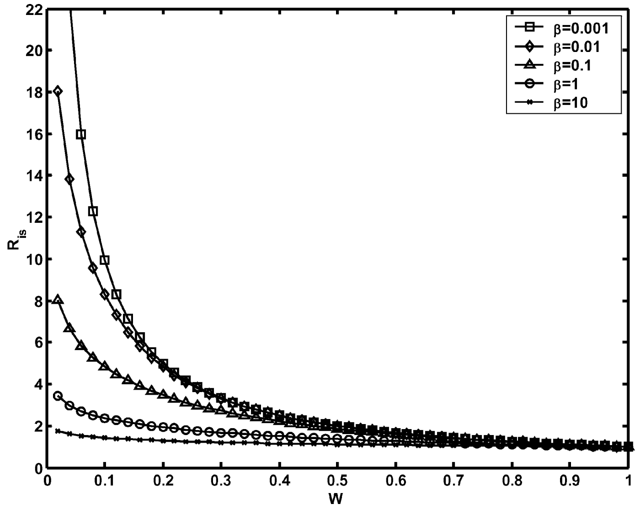

Fig. 2 Nondimensional thermal resistance for insulated right and isothermal bottom surfaces.

which is attributed to the normalization and definition of the thermal resistance.

The effect of convective sink is demonstrated in Fig. 3 for case 2 (insulated right and convective bottom surfaces). The solution is presented graphically in Fig. 3 for several values of the Biot number and nondimensional variable β. Biot numbers based on the vertical plate thermal conductivity k_y range from 0.001 to 10. Results indicated that the thermal resistance is independent of the Biot number when it is less than 0.001 or larger than 10. Furthermore, one can see that the thermal resistance for $\beta = 10$ is the same for all Biot numbers and may not represent a realistic value for any engineering application. This case was included in Fig. 3 as a baseline to provide the user with the flexibility to interpret the data between $\beta = 1$ and 10 if needed.

In space applications, electronic cooling is usually conducted in an isothermal environment. Figure 4 presents the nondimensional thermal resistance for case 3 (isothermal right and isothermal bottom surfaces).

The expressions for temperatures and thermal resistance depend on several dimensionless geometric and thermal parameters. Based on the dimensionless physical parameters, one can determine the effective resistance in the heat spreading bodies.

Thermal resistance exhibits the same trend with respect to the heated length ratio of the plate regardless of the plate aspect ratio,

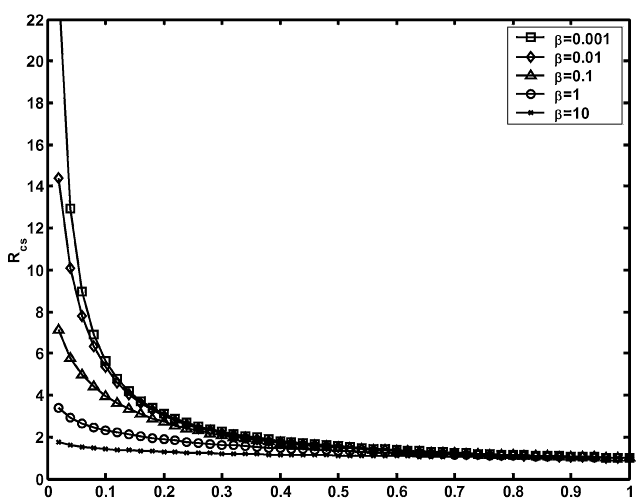

Fig. 3a Biot number effect on nondimensional thermal resistance for insulated right and convective bottom surfaces, *Bi* = 0.001.

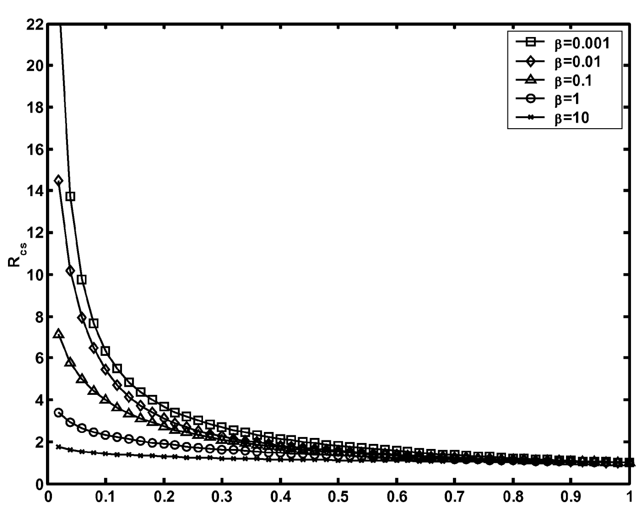

Fig. 3b Biot number effect on nondimensional thermal resistance for insulated right and convective bottom surfaces, *Bi* = 0.01.

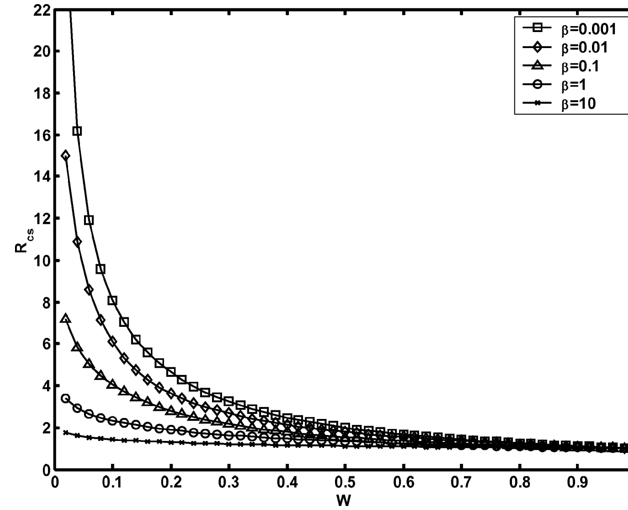

Fig. 3c Biot number effect on nondimensional thermal resistance for insulated right and convective bottom surfaces, *Bi* = 0.1.

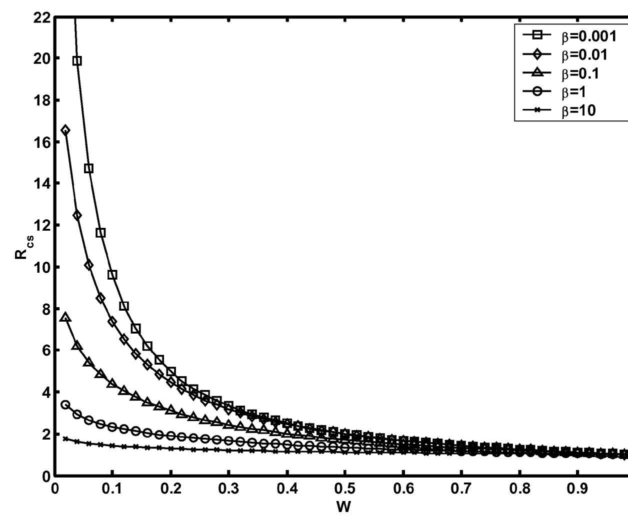

Fig. 3d Biot number effect on nondimensional thermal resistance for insulated right and convective bottom surfaces, *Bi* = 1.

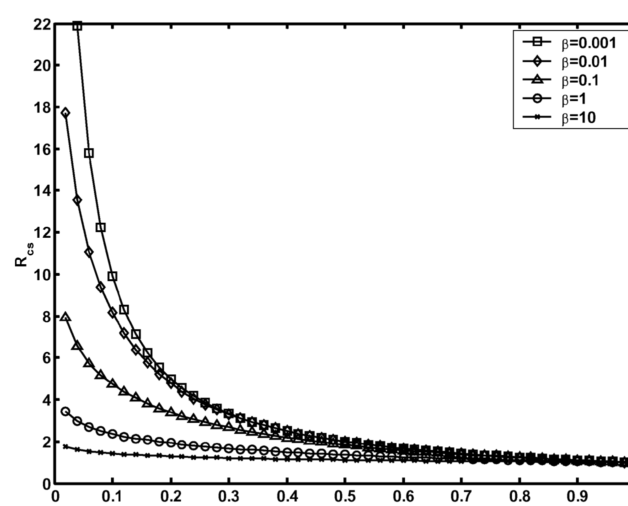

Fig. 3e Biot number effect on nondimensional thermal resistance for insulated right and convective bottom surfaces, *Bi* = 10.

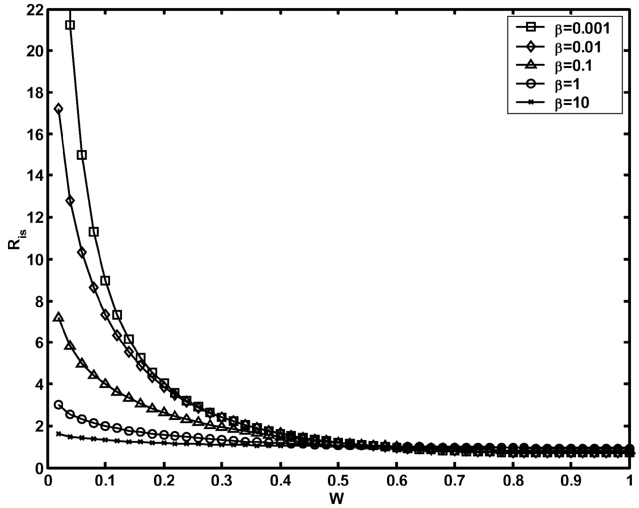

Fig. 4 Nondimensional thermal resistance for isothermal right and isothermal bottom surfaces.

Biot number, or boundary condition. The generalized resistance is defined as the ratio of the dimensional resistance to a reference resistance. This resistance ratio approaches unity as the heated length a approaches the spreader width b.

More simply, the resistance ratio approaches unity as W approaches unity. As the heated length ratio W becomes smaller, the resistance ratio increases. This means that the actual thermal resistance deviates from that of a fully heated plate. Similarly, as the plate aspect ratio H becomes smaller, the plate resistance ratio becomes larger for any given value of W. This effect is the result of the heat spreading capability of the plate. A comparison of thermal resistance for the orthotropic properties of graphite to pure aluminum shows that, for a given thickness of material, the large vertical conductance of aluminum has a lower resistance. Yet, as we will see from the engineering examples, a strong orthotropic conductance will spread heat over a larger surface area. Consequently, a smaller aspect ratio graphite spreader will provide the same through resistance as an isotropic plate of aluminum.

Engineering Applications

Several examples will be shown in this section to demonstrate the applicability of the graphs for the determination of the thermal resistance and temperature drop across the electronic box mounting plate and semiconductor chips thermal spreaders.

Example 1: Insulated Right and Isothermal Bottom Surfaces

Consider the case of a semiconductor chip conducting heat into an aluminum thermal heat sink with dimensions 0.02 by 0.02 by 0.01 m and a 20-W power input over an area of 0.004 by 0.001 m. The thermal designer is interested in replacing the aluminum heat sink with a pure K1100 cyanate graphite epoxy spreader or a K1100 C–C matrix graphite spreader. The engineer will use the same footprint as the aluminum spreader but will reduce the K1100 thickness until the same ratio of the minimum thermal resistance to the reference thermal resistance is achieved.

The reference thermal resistance R_r can be calculated by the use of Eq. (10), and the generalized dimensionless thermal resistance R_{is} is found in Fig. 2. Once the information is available, the thermal resistance of the plate, $\bar{R}_{is}$, can be determined with Eq. (11), and the temperature drop across the plate can easily be obtained by multiplication of the power input with the resulting thermal resistance. The results are listed in Table 3.

The thickness of the original aluminum plate is 0.01 m. The thickness of the K1100 graphite epoxy plate is 0.000868 m, and the K1100 graphite/C–C plate is 0.003428 m. The mass savings is 94% for the K1100 graphite epoxy plate and 77% for the K1100 graphite/C–C plate.

Table 3 Example 1: insulated right and isothermal bottom surfaces

Material	Reference resistance R_r	Generalized resistance R_{is}	Dimensional resistance $\bar{R}_{is}$	Temperature change °C
Pure Al	0.002	2.77	0.006	0.125
K1100 graphite/epoxy	0.022	2.77	0.06	1.2
K1100 graphite/C–C	0.004	2.77	0.012	0.24

Table 4 Example 2: insulated right and convective bottom surfaces

Biot number Bi	Reference resistance R_r	Generalized resistance R_{cs}	Dimensional resistance $\bar{R}_{cs}$	Temperature change °C
0.001	$5.42E-5$	1.1057	$5.99E-5$	0.006
0.01	$5.42E-5$	1.1147	$6.04E-5$	0.006
1	$5.42E-5$	1.1164	$6.05E-5$	0.006

Table 5 Example 3: isothermal right and isothermal bottom surfaces

Material	Reference resistance R_r	Generalized resistance R_{is}	Dimensional resistance $\bar{R}_{is}$	Temperature change °C
Pure Al	0.0002	4.03	0.0006	0.02
K1100 graphite/epoxy	0.0004	4.02	0.0016	0.06
K1100 graphite/C–C	0.0001	4.02	0.0005	0.02

Example 2: Insulated Right and Convective Bottom Surfaces

Let us consider the case of a electronic box thermal spreader that is made of K1100 graphite/C–C with dimensions 0.5 by 0.5 by 0.006 m and a 100-W power input over an area of 0.45 by 0.005 m. It is assumed that the incoming energy is either radiating to a medium or convecting to an environment with Biot numbers that range from 0.001 to 1. To design an effective thermal spreader, the thermal designer is interested in the actual minimum resistance and the temperature drop through the plate thickness, as well as the convective effect.

The calculation procedure is the same as example 1. The reference thermal resistance R_r can be calculated by the use of Eq. (10) and the generalized dimensionless thermal resistance R_{cs} is found in Fig. 3. Once the information is available, the thermal resistance of the plate $\bar{R}_{cs}$ can be determined with Eq. (13), and the temperature drop across the plate can easily be obtained by multiplication of the power input with the resulting thermal resistance. The results are given in Table 4. Similar results were calculated for pure aluminum and K1100 graphite epoxy spreaders. The reference resistance in each case was kept constant, and the thickness of the doubler plate was determined. The graphite composite materials showed a weight savings of over 90% compared to an aluminum heat spreader.

Example 3: Isothermal Right and Isothermal Bottom Surfaces

Consider the spreader design for a box that is 0.127 by 0.127 m. There are 20 W of heat applied along a 0.0381 by 0.005 m section near the center of the box. In this case, it was assumed that the thickness of the materials are known. The pure aluminum is 0.006 m, the K1100 graphite epoxy is 0.00015 m, and the K1100 C–C thickness is 0.00102 m. Each spreader is designed to extend beyond the edge of the box by 0.063 m. This distance is sufficient to spread the heat fully for each of the candidate thicknesses. The generalized resistance is identical for each of the spreaders.

The reference thermal resistance R_r can be calculated by the use of Eq. (10), and the generalized dimensionless thermal resistance R_{is} is found in Fig. 4. Once the information is available, the thermal resistance of the plate $\bar{R}_{is}$ can be determined with Eq. (11), and the temperature drop across the plate can easily be obtained by multiplication of the power input with the resulting thermal resistance. The results are listed in Table 5.

As before, the weight savings with a highly conductive othotropic K1100 graphite spreader is considerable. Performance is not compromised by the low conductance in the k_y direction. Thickness can be reduced because the heat conducts very well in the orthotropic direction.

Summary

The solution to the generalized thermal resistance of orthotropic thermal spreaders subjected to a uniform heat flux was solved. The configuration investigated simulates the cooling of an integrated circuit device. For convenience, the data are presented as a series of plots of the thermal resistance as a function of the relative width of the applied heat flux region, the length-to-thickness conductivity ratio, and the Biot number. At values of the applied heat flux width equal to the spreader width, the thermal resistance tended toward unity, as expected.

Examples of the heat spreader design show the benefit of the generalized solution. Thermal engineers can optimize the design of the spreader and make intelligent choices for materials and spreader dimensions by the use of simple tables or plots of the generalized solutions.

References

[1]Lam, T. T., Santiago, J. G., and Fischer, W. D., "Minimum Thermal Resistance of a Convectively Cooled Orthotropic Plate with a Uniform Heat Flux," *Advances in Electronic Packaging*, Vol. 2, American Society of Mechanical Engineers, Fairfield, NJ, 1997, pp. 1947–1953.

[2]Lam, T. T., and Fischer, W. D., "Thermal Resistance in Rectangular Orthotropic Heat Spreaders," *Advances in Electronic*, ASME EEP-Vol. 26-1, American Society of Mechanical Engineers, Fairfield, NJ, 1999, pp. 891–898.

[3]Yovanovich, M. M., Muzychka, Y. S., and Culham, J. R., "Spreading Resistance of Isoflux Rectangles and Strips on Compound Flux Channels," *Journal of Thermophysics and Heat Transfer*, Vol. 13, No. 4, 1999, pp. 495–500.

[4]Kadambi, V., and Abuaf, N., "An Analysis of the Thermal Response of Power Chip Packages," *IEEE Transactions on Electron Devices*, Vol. ED-32, No. 6, 1995, pp. 1024–1033.

[5]Kennedy, D. P., "Spreading Resistance in Cylindrical Semiconductor Devices," *Journal of Applied Physics*, Vol. 31, No. 8, 1960, pp. 1490–1497.

[6]Lindsted, R. D., and Surty, R. J., "Steady-State Junction Temperatures of Semiconductor Chips," *IEEE Transactions on Electron Devices,* Vol. ED-19, No. 1, 1972, pp. 41–44.

[7]Schneider, G. E., Yovanovich, M. M., and Cane, R. L. D., "Thermal Resistance of a Convectively Cooled Plate with Nonuniform Applied Flux," *Journal of Spacecraft and Rockets*, Vol. 17, No. 4, 1978, pp. 372–376.

[8]Venkataraman, N. S., Pontes Cardoso, H., and de Oliveira Filho, O. B., "Thermal Resistance of Convectively Cooled Plates with Applied Heat Flux and Internal Heat Generation," *Acta Astronautics*, Vol. 9, No. 10, 1982, pp. 613–620.

[9]Carslaw, H. S., and Jaeger, J. C., *Conduction of Heat in Solids*, Oxford Univ. Press, Oxford, 1959, p. 42.

[10]Ozisik, M. N., *Heat Conduction*, 2nd ed., Wiley, New York, 1993, pp. 617–653.

[11]Krane, M. J. M., "Constriction Resistance in Rectangular Bodies," *Journal of Electronic Packaging*, Vol. 113, Dec. 1991, pp. 392–396.